Problem based on
Cartesian Sub-tangent, Sub-normal, tangent, normal

Sub-tangent $= \frac{y}{y_1}$ Sub-normal $= y y_1$
length of sub-tangent $= \frac{y}{y_1}\sqrt{1+y_1^2}$
length of normal $= y\sqrt{1+y_1^2}$
Where $y_1 = \frac{dy}{dx}$

Problem :→ Show that the subtangent at any point on the curve

$$x^m y^n = a^{m+n}$$ varies as

Solution abscissa.
Since subtangent $= \frac{y}{y_1}$

Now by the question
Equation of the curve is
$$\therefore x^m y^n = a^{m+n} \quad \text{——— (1)}$$
Diff. with respect to x

$$m x^{m-1} y^n + n x^m y^{n-1} \frac{dy}{dx} = 0$$

$$n x^m y^{n-1} \frac{dy}{dx} = -m x^{m-1} y^n$$

$$\frac{dy}{dx} = -\frac{m}{n}\frac{y}{x}$$

$$\therefore \text{Subtangent} = y/y_1 = \frac{y}{\frac{dy}{dx}}$$

$$= \frac{n y x}{-m y} = -\frac{n}{m} x$$

$\Rightarrow$ Subtangent $\propto x$
thus subtangent at any point of the curve varies as the abscissa. proved

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Problem 1-7
Show that in the curve $y = b e^{-a/x}$ the subtangent varies as the square of the abscissa.

Solution - Since Subtangent $= \frac{y}{y_1} = \frac{y}{\frac{dy}{dx}}$ —(1)

Here Equation of curve is
$y = b e^{-a/x}$

Diff. with respect to x
$$\frac{dy}{dx} = b \frac{d\, e^{-a/x}}{d(-a/x)} (-a) \frac{d\frac{1}{x}}{dx}$$

$$= b e^{-\frac{a}{x}} (-a)\left(-\frac{1}{x^2}\right)$$

$$= a b e^{-\frac{a}{x}} \cdot \frac{1}{x^2}$$

Now subtangent
$$= \frac{y}{dy/dx}$$

$$= \frac{\cancel{b} \cancel{e^{-\frac{a}{x}}}}{a \cancel{b} \cancel{e^{-\frac{a}{x}}} \cdot \frac{1}{x^2}}$$

$$= \frac{x^2}{a}$$

Hence subtangent
$\Rightarrow$ Subtangent $= \frac{x^2}{a}$
Subtangent to the given curve is varies as square of the abscissa.

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Show that in the curve
$by^2 = (a+x)^3$
the square of the subtangent varies as the normal

Solution →
Since subtangent $= \dfrac{y}{\frac{dy}{dx}}$

and sub-normal $= y \cdot \dfrac{dy}{dx}$

we are to show
square of the subtangent ∝ subnormal

i.e. $\dfrac{(\text{square of subtangent})^2}{\text{sub normal}} = \text{const.}$

Here equation of the curve is

$$by^2 = (a+x)^3$$

$$2by\frac{dy}{dx} = 3(a+x)^2$$

$$\frac{dy}{dx} = \frac{3(a+x)^2}{2by}$$

∴ Subtangent $= \dfrac{y}{y} = \dfrac{y}{\frac{a+x}{by}}$

$= \dfrac{by^2}{a+x}$

Sub normal y

Subtangent

$= \frac{y}{y_1}$

$= \frac{y \times 2by}{3(a+x)^2}$

$= \frac{2by^2}{3(a+x)^2} = \frac{2(a+x)^3}{3(a+x)^2} = \frac{2}{3}(a+x)$

$\because by^2 = (a+x)^3$

Subnormal $= y \cdot y_1$

$= \cancel{y} \times \frac{3(a+x)^2}{2b\cancel{y}}$

$= \frac{3}{2b}(a+x)^2$

Now $\frac{(\text{Subtangent})^2}{\text{Subnormal}} = \frac{4 \times 2b\,\cancel{(a+x)^2}}{9 \times 3\,\cancel{(a+x)^2}}$

$= \frac{8b}{27} =$ constant.

Therefore $(\text{Subtangent})^2 \propto \text{Subnormal}$

Show that at any point of the hyperbola $xy = c^2$ the subtangent varies as the abscissa and the sub-normal varies as the cubes of the ordinate of the point of contact.

Solution :→ Here equation of the curve is

$xy = c^2$

$y + x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Now Subtangent $= \frac{y}{y_1} = -\frac{y}{y/x} = -x$.

Now subtangent $= -\frac{x}{x} = -1 =$ const.

and ~~subnormal x~~ $\Rightarrow$ subtangent $\propto$ abscissa the Square of the Subtangent varies as the normal

Solution →

Since Subtangent $= \dfrac{y}{\frac{dy}{dx}}$

and Sub-Normal $= y \cdot \dfrac{dy}{dx}$

We are to show

Square of the Subtangent ∝ Subnormal

i.e $\dfrac{(\text{Square of subtangent})^2}{\text{Sub Normal}} = \text{Const.}$

Here Equation of the curve is

$$by^2 = (a+x)^3$$

$$2by \frac{dy}{dx} = 3(a+x)^2$$

$$\frac{dy}{dx} = \frac{3(a+x)^2}{2by}$$

$\therefore$ Subtangent $= \dfrac{y}{y} = \dfrac{y}{\frac{a+x}{by}}$

$$= \frac{by^2}{a+x}$$

Subnormal

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Subtangent

$$= \frac{y}{y_1}$$

$$= \frac{y \times 2by}{3(a+x)^2}$$

$$= \frac{2by^2}{3(a+x)^2} = \frac{2(a+x)^3}{3(a+x)^2} = \frac{2}{3}(a+x)$$

$$\because by^2 = (a+x)^3$$

Sub Normal $= y \cdot y_1$

$$= \cancel{y} \times \frac{3(a+x)^2}{2b\cancel{y}}$$

$$= \frac{3}{2b}(a+x)^2$$

Now $\frac{(\text{Sub tangent})^2}{\text{Sub normal}} = \frac{4 \times 2b\,\cancel{(a+x)^2}}{9 \times 3\,\cancel{(a+x)^2}}$

$$= \frac{8b}{27} = \text{constant}$$

Therefore $(\text{Subtangent})^2 \propto$ SubNormal

Show that at any point of the hyperbola $xy = c^2$ the subtangent varies as the abscissa and the sub-normal varies as the cubes of the ordinate of the point of contact.

Solution :→ Here equation of the curve is

$$xy = c^2$$

$$y + x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

Now Subtangent $= \frac{y}{y_1} = -\frac{y}{y/x} = -x$

Now $\frac{\text{subtangent}}{x} = -\frac{x}{x} = -1 = \text{const.}$

and ~~subtangent/x~~ ⇒ subtangent $\propto$ abscissa

Sub Normal $= y \cdot y_1$

$= y \cdot (-y/x)$

But $xy = c^2$

$\Rightarrow x = \dfrac{c^2}{y}$

$= -\dfrac{y^2 \times y}{c^2} = -\dfrac{y^3}{c^2}$

Now $\dfrac{\text{Sub Normal}}{y^3} = -\dfrac{y^3}{c^2 \times y^3} = -\dfrac{1}{c^2} = \text{Const.}$

Shows that Sub-Normal varies as the cubes of ordinate.